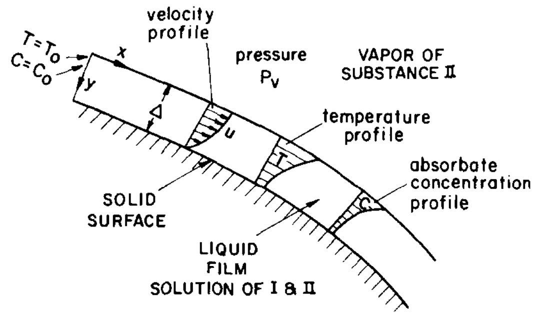

FIG. 2. Variation of C_{Nu} with Z for different Le and λ.

It is evident that the Sherwood number is independent of K_1 and Z and has the same value as in the case with no interdiffusion. The Nusselt number deviates from the $Z = 0$ case by a factor equal to the term in square brackets in equation (23). This deviation factor is defined as:

$$C_{Nu} = Nu_x \Big/ \left(\frac{1}{\sqrt{\pi}}\sqrt{\frac{u_0 x}{\alpha}}\right) = \frac{\lambda K_1}{\sqrt{Le}\,(1-K_1)}. \qquad (24)$$

Figure 2 describes the variation of C_{Nu} with Z for several values of Le and λ. Increasing Z decreases the value of the Nusselt number, relative to its value in the absence of interdiffusion. It is also clear that the larger λ and lower Le, the lower the value of C_{Nu}. It should be noted that larger λ and lower Le lead to a higher temperature at the vapor–liquid interface. Thus, while the heat transfer coefficient improves with decreasing λ and increasing Le, the overall heat effect associated with the absorption process is reduced under this condition.

REFERENCES

1. V. E. Nakoryakov and N. I. Grigor'eva, Combined heat and mass transfer during absorption in drops and films, *Inzh.-fiz. Zh.* **32**, 399–405 (1977).
2. N. I. Grigor'eva and V. E. Nakoryakov, Exact solution of combined heat and mass transfer problem during film absorption, *Inzh.-fiz. Zh.* **33**, 893–898 (1977).
3. S. M. Yih and R. C. Seagrave, Mass transfer in laminar falling liquid films with accompanying heat transfer and interfacial shear, *Int. J. Heat Mass Transfer* **23**, 749–758 (1980).
4. V. E. Nakoryakov and N. I. Grigor'eva, Calculation of heat and mass transfer in non-isothermal absorption in the entrance region of a falling film, *Teor. Osn. Khim. Tekhnol.* **14**, 483–488 (1980).
5. G. Grossman, Simultaneous heat and mass transfer in film absorption under laminar flow, *Int. J. Heat Mass Transfer* **26**, 357–371 (1983).
6. G. Grossman, Simultaneous heat and mass transfer in absorption of gases in turbulent liquid films, *Int. J. Heat Mass Transfer* **27**, 2365–2376 (1984).
7. J. O. Hirschfelder, C. F. Curtiss and R. B. Bird, *Molecular Theory of Gases and Liquids*. John Wiley, New York (1954).
8. R. Higbie, The rate of absorption of a pure gas into a still liquid during short periods of exposure, *Trans. A.I.Ch.E.* **31**, 365 (1935).

Int. J. Heat Mass Transfer. Vol. 30, No. 1, pp. 208–211, 1987
Printed in Great Britain

0017-9310/87 $3.00+0.00
Pergamon Journals Ltd.

Thermal convection in a porous medium subject to transient heating and rotation

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(*Received* 30 *December* 1985 *and in final form* 13 *May* 1986)

1. INTRODUCTION

THE DESCRIPTION of thermal convection in a porous medium is mainly based on Darcy's law, which includes boundary and inertial effects [1]. The global effects of the fluid are derived by using volume average techniques [2–3]. Linear stability theory of the steady state may predict the onset of thermal convection at the marginal state [4], while nonlinear stability theory can differentiate the flow patterns and determine the subcritical instability [5–7]. Empirically, the cell patterns, first appearing at the marginal state, continue to manifest the same patterns in the weakly nonlinear stability state [8]. Foster [9] treats the thermal convection of transient state as an initial value problem. Amplification theory, requiring the empirically determined initial conditions, is applied to predict the critical time of thermal convection [10–13].

The onset of thermal convection of both steady and transient states in a porous medium rotating with an angular frequency is considered. Both upper and lower boundaries are free and fixed at a constant temperature T_0. The initial temperature distribution is nonlinear and is increased from the below at a constant rate c.

2. FORMULATION AND ANALYSIS

The dimensionless governing equations and conditions of the perturbed state, assuming the Boussinesq approximation, are

$$\left(\frac{\partial}{\partial t}+p_r\frac{\delta}{K}-p_r\nabla^2\right)^2\nabla^2 w = p_r\left(\frac{\partial}{\partial t}+p_r\frac{\delta}{K}-p_r\nabla^2\right)\nabla_1^2\theta - T_a D^2 w \qquad (1)$$

$$\left(\frac{\partial}{\partial t}-\nabla^2\right)\theta = R(-DT)w \qquad (2)$$

$$\left(\frac{\partial}{\partial t}-D^2\right)T = 0 \qquad (3)$$

$$w = D^2 w = D^4 w = \theta = 0, \quad \text{at} \quad z = 0, 1 \qquad (4)$$

$$T(0,t) = ct, \quad T(1,t) = 0 \quad \text{and} \quad T(z,0) = \frac{(1-z)z}{2}. \qquad (5)$$

The solution for the basic temperature, from equations (3)

NOMENCLATURE

a	wavenumber, $(k_1^2+k_2^2)^{1/2}$
$A_m(t)$	amplitude function of transient state
c	heating rate
C	specific heat
D	$\partial/\partial z$
g	gravity
H	internal heat source setting up the initial nonlinear temperature profile
k	thermal conductivity
K	permeability
L	depth of layer
p_r	Prandtl number, ν_f/α_e
R	Rayleigh number, $\rho_e g L^5 H\alpha/\alpha_e^3$
t	time
T	basic temperature
T_0	initial temperature of upper and lower boundaries
T_a	Taylor number, $4\Omega^2L^4/\alpha_e^2$
w	vertical velocity
x, y, z	Cartesian coordinates.

Greek symbols

α	coefficient of thermal expansion
α_e	effective thermal diffusivity, $[k_f\delta+k_s(1-\delta)]/[\rho_f C_f\delta+\rho_s C_s(1-\delta)]$
δ	porosity
δ_{mr}	delta function
∇_1^2	$\partial^2/\partial x^2+\partial^2/\partial y^2$
θ	temperature perturbation
ν	kinematic viscosity
ρ	density
ρ_e	$\rho_f C_f\delta/[\rho_f C_f\delta+\rho_s C_s(1-\delta)]$
Ω	angular frequency.

Subscripts

c	critical
f	fluid
s	solid.

and (5), is

$$T(z,t) = -c(z-1)t+\frac{2}{\pi^3}\sum_{n=1}^{\infty}\frac{\sin n\pi z}{n^3} \times\{[1-(-1)^n+c]e^{-n^2\pi^2 t}-c\}. \quad (6)$$

The equation for w may be derived by eliminating θ in equations (1) and (2), then the conditions in equation (4) allow the normal mode for w

$$w(x,y,z,t) = \sum_{m=1}^{\infty} A_m(t)\sin m\pi z\, e^{i(k_1x+k_2y)}. \quad (7)$$

Multiplying $\sin r\pi z$ on both sides of the w equation and integrating on z from 0 to 1, the amplitude equations then become

$$A_r'''+A_r''[2p_r\alpha(r)+\beta(r)]+A_r'\left\{p_r\alpha(r)[p_r\alpha(r)+2\beta(r)] + T_a\frac{r^2\pi^2}{\beta(r)}\right\}+A_r[p_r^2\alpha^2(r)\beta(r)+T_a r^2\pi^2] = -\frac{2}{\beta(r)}Ra^2 \times\left\{\sum_{m=1}^{\infty}A_m[J_{mr}+p_r m^2r^2 I_{mr}]+\sum_{m=1}^{\infty}A_m' I_{mr}\right\},\quad r=1,2,\ldots \quad (8)$$

where

$$A_r' = \frac{dA_r}{dt},\quad \alpha(r) = r^2\pi^2+a^2+\frac{\delta}{K},\quad \beta(r) = r^2\pi^2+a^2$$

and

$$I_{mr} = \int_0^1 \frac{\partial T}{\partial z}\sin m\pi z\sin r\pi z\,dz = \begin{cases} \dfrac{[1-(-1)^{m-r}+c]e^{-(m-r)^2\pi^2 t}-c}{2\pi^2(m-r)^2} - \dfrac{[1-(-1)^{m+r}+c]e^{-(m+r)^2\pi^2 t}-c}{2\pi^2(m+r)^2}, & m\neq r \\ -\dfrac{1}{2}ct-\dfrac{c\cdot e^{-(m+r)^2\pi^2 t}-c}{2\pi^2(m+r)^2}, & m=r \end{cases} \quad (9)$$

$$J_{mr} = \int_0^1\left\{\left[\frac{\partial}{\partial t}+p_r\frac{\delta}{K}-p_r(D^2-a^2)\right]\frac{\partial T}{\partial z}\right\}\sin m\pi z\sin r\pi z\,dz =$$

$$\begin{cases} \dfrac{1}{2\pi^2(m-r)^2}\{[1-(-1)^{m-r}+c]e^{-(m-r)^2\pi^2 t}-c\} \cdot\left\{-(m-r)^2\pi^2+p_r\left[(m-r)^2\pi^2+\dfrac{\delta}{K}+a^2\right]\right\} \\ \quad -\dfrac{1}{2\pi^2(m+r)^2}\{[1-(-1)^{m+r}+c]e^{-(m+r)^2\pi^2 t}-c\} \cdot\left\{-(m+r)^2\pi^2+p_r\left[(m+r)^2\pi^2+\dfrac{\delta}{K}+a^2\right]\right\}, & m\neq r \quad (10) \\ \dfrac{1}{2}\left[-3c-p_r\left(\dfrac{\delta}{K}+a^2\right)ct\right]-\dfrac{1}{2\pi^2(m+r)^2} \times\{c\cdot e^{-(m+r)^2\pi^2 t}-c\}\cdot\left\{-(m+r)^2\pi^2 +p_r\left[(m+r)^2\pi^2+\dfrac{\delta}{K}+a^2\right]\right\}, & m=r. \end{cases}$$

3. STEADY STATE

Analysis of the linear stability of the steady state is, assuming all the time derivatives in equation (8) to be zero, described by

$$\sum_{m=1}^{\infty}A_m\left\{\delta_{mr}[p_r^2\alpha^2(r)\beta(r)+T_a r^2\pi^2] +\frac{2}{\beta(r)}Ra^2[J_{mr}+p_r m^2\pi^2 I_{mr}]\right\} = 0. \quad (11)$$

In order to obtain nontrivial solutions for the characteristic problem, the determinant of equation (11) must vanish. The Rayleigh numbers R as functions of the wavenumber a are solved for various values of p_r, δ/K and T_a. The critical Rayleigh number R_c with respect to the critical wave number a_c marks the marginal state. For $p_r = 1$, $\delta/K = 0$ and $T_a = 0$, we have numerically solved $R_c = 16992$ and $a_c = 3.02$ as obtained previously [14]. Critical Rayleigh numbers R_c and wavenumbers a_c as functions of p_r, δ/K and T_a are shown in Figs. 1 and 2, respectively.

For small values of p_r, R_c and a_c increase with T_a rapidly, when δ/K is small, and slowly, when δ/K becomes large. However, R_c and a_c increase with δ/K for moderate values of T_a and decrease with δ/K for very large values of T_a. For

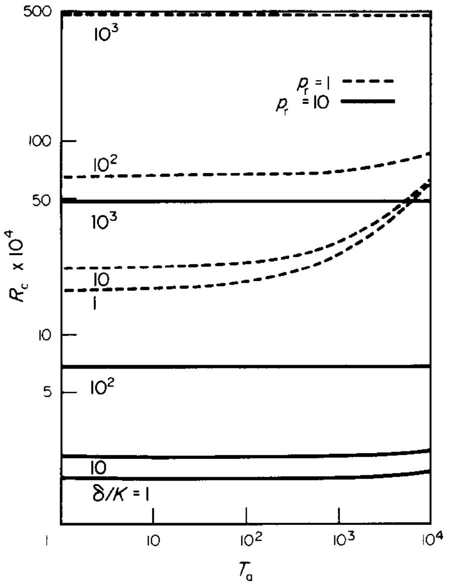

FIG. 1. Variation of critical Rayleigh number R_c with Taylor number T_a.

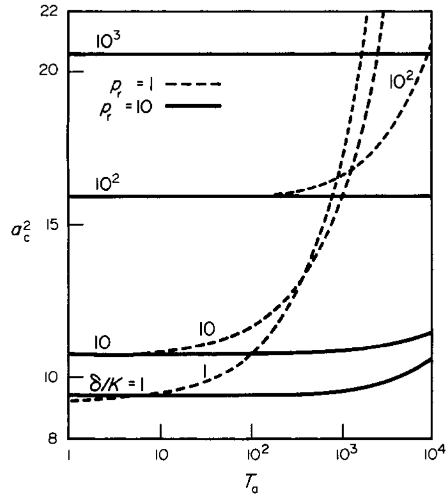

FIG. 2. Variation of critical wavenumber a_c with Taylor number T_a.

large values of p_r, the increasing of R_c and a_c with T_a are relatively insignificant. The results show that R_c and a_c increase with p_r very sensitively.

4. TRANSIENT STATE

The thermal convection of the transient state is governed by equation (9). The involved parameters are R, a^2, p_r, δ/K, T_a and c. The Rayleigh number R may be less than or equal to that of the marginal state R_c. The system of ordinary differential equations is numerically solved by using a fourth-order Runge–Kutta–Gill method. Foster [9] suggests that all the amplitude disturbances may assume the same value as the initial conditions. We adopt $A_r(0) = 1$, $A_r'(0) = 0$ and $A_r''(0) = 1$. The growth factor of average velocity disturbances is defined as

$$\bar{w}(t) = \left[\int_0^1 w^2(z,t)\,\mathrm{d}z \Big/ \int_0^1 w^2(z,0)\,\mathrm{d}z \right]^{1/2}. \quad (12)$$

The time for $\bar{w}(t)$ to reach a value of 1000 is chosen as the critical time t_c for the onset of transient thermal convection. The choice of value 1000 is not unique, however, a larger value than 1000 does not change the critical time significantly [15]. The dimensional time scale used is about 10^4–10^5 s.

For given values of R, p_r, δ/K, T_a and c, the wavenumber with respect to the minimum critical time is called the critical wavenumber a_c. Thermal convection, appearing with the flow patterns of critical wavenumber, will grow quickly. It is worth mentioning that thermal convection does not necessarily appear with the pattern of critical wavenumber [8]. Taylor–Proudman theory [4] predicts that the rotation effect alone acts as a stabilizing factor. A large thermal diffusivity means effective conduction of thermal energy throughout the fluid layer, and the tendency for the fluid to become destabilizing is, then, reduced. The frictional and form drags, due to a large porosity or small permeability, may suppress the possible thermal convection. Therefore, larger values of p_r, δ/K or T_a will extend the critical time significantly.

The critical time t_c as a function of the Prandtl number p_r is shown in Fig. 3. The values of parameters are $R = 17{,}619$, $\delta/K = 100$, $c = 1$ and the critical wavenumbers. The critical time decreases with p_r sensitively, when $p_r < 10$, and insignificantly, when $p_r > 10$. The extension of critical time with the Taylor number is very obvious.

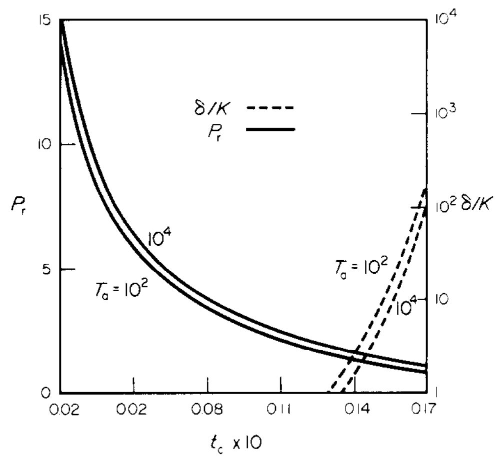

FIG. 3. Variation of critical time t_c with Prandtl number p_r and the ratio of porosity to permeability δ/k.

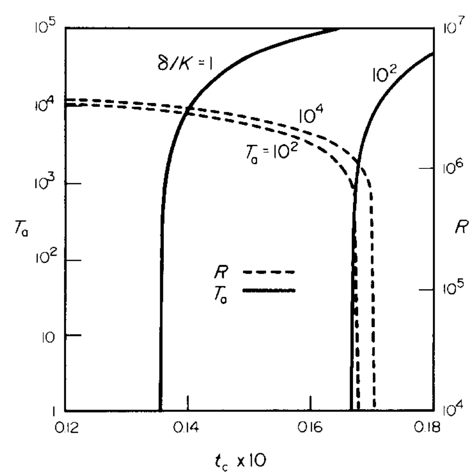

FIG. 4. Variation of critical time t_c with Rayleigh number R and Taylor number T_a.

The critical time t_c as a function of the ratio of porosity to permeability δ/K is also shown in Fig. 3. The values of parameters are $R = 17{,}619$, $p_r = 1$, $c = 1$ and the critical wavenumbers. The critical time increases significantly with δ/K.

The critical time t_c as a function of the Rayleigh number R is shown in Fig. 4. The values of parameters are $p_r = 1$, $\delta/K = 100$, $c = 1$ and the critical wavenumbers. When R is less than R_c of the marginal state, the transient heating from below is the main factor causing the thermal convection and t_c varies slowly with R. When R is greater than or equal to R_c of the marginal state, the fluid flows are in the weak nonlinear state of finite amplitude. The higher value of R corresponds to a more destabilizing transient state, the decrease of t_c with R is very significant.

In the discussions above, the Taylor number acts as a stabilizing factor and extends the critical time effectively. The critical time t_c as a function of the Taylor number T_a is also shown in Fig. 4. The values of parameters are $R = 17{,}619$, $p_r = 1$, $c = 1$ and the critical wavenumbers. The increase of t_c with T_a is very small, when $T_a < 10^4$.

REFERENCES

1. K. Vafai and C. L. Tien, Boundary and inertial effects on flow and heat transfer in porous medium, *Int. J. Heat Mass Transfer* **24,** 195–203 (1974).
2. J. C. Slattery, *Momentum, Energy and Mass Transfer in Continua*. McGraw-Hill, New York (1972).
3. S. Whitaker, Advances in theory of fluid motion in porous media, *J. Ind. Engng Chem.* **61,** 14–28 (1969).
4. S. Chandrasekhar, *Hydrodynamic and Hydromagnetics Stability*. Oxford University Press, London (1961).
5. R. Krishnamurti, Finite amplitude convection with changing mean temperature, *J. Fluid Mech.* **33,** 445–455 (1968).
6. F. H. Busse and N. Riahi, Nonlinear convection in a layer with nearly insulating boundaries, *J. Fluid Mech.* **96,** 243–256 (1980).
7. N. Riahi, Nonlinear convection in a porous medium with internal heat sources, *Int. J. non-linear Mech.* **19,** 469–478 (1984).
8. J. T. Stuart, The nonlinear mechanics of hydrodynamic stability, *J. Fluid Mech.* **56,** 353–374 (1958).
9. T. D. Foster, Stability of homogeneous fluid cooled uniformly from above, *Phys. Fluids* **8,** 1249–1257 (1965).
10. M. Kaviany, Thermal convective instabilities in a porous medium, *J. Heat Transfer* **106,** 137–142 (1984).
11. I. E. Davenport and C. J. King, The onset of natural convection from time-dependent profiles, *Int. J. Heat Mass Transfer* **17,** 69–76 (1974).
12. M. Kaviany, Onset of thermal convection in a fluid layer subjected to transient heating from below, *J. Heat Transfer* **106,** 817–823 (1984).
13. P. M. Gresho and R. L. Sani, The stability of a fluid layer subjected to a step change in temperature transient vs frozen time analysis, *Int. J. Heat Mass Transfer* **14,** 207–221 (1974).
14. F. A. Kulacki and R. J. Goldstein, Hydrodynamic instability in fluid layers with uniform volumetric energy source, *Appl. scient. Res.* **31,** 81–109 (1975).
15 T. D. Foster, Onset of manifest convection in a layer of fluid with a time-dependent surface temperature, *Phys. Fluids* **12,** 2482–2487 (1969).